A Framework for Faster Key Search Using Related-key Higher-order Differential Properties

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2. Higher-order Differential Characteristic

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Related-key Cryptanalysis

- In the **related key attack** the attacker aims to derive some information about the key based on an assumption that he is capable of obtaining the encryption of plaintexts or the decryption of ciphertexts under several keys so that the **relation** between the keys is **chosen** by the **attacker**.

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- **Related-key** model is usually **not** a **realistic** model hence some designers do not take this type of attacks into account.
- A natural question is how well the **related-key** distinguishers can lead to **serious vulnerabilities** in practice?

Deterministic Related-key Differential Cryptanalysis

- For the first time Deffie and Helmann introduced a deterministic relation between the encryption of DES under the **key K** and **related key** $\bar{K}$ namely for every K and P if $C = E_K(P)$ then $\bar{C} = E_{\bar{K}}(\bar{P})$.

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- We denote the set $\mathcal{K}_0$ as the subspace of all n-bit keys in which the LSB is zero: $\mathcal{K}_0 = \{K \in \mathbb{F}_2^n, LSB(K) = 0\}$.

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   - Compute $E_K(P)$; if it is equal to C, return K.
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This property **decrease the security** of DES with **one bit**.

- Hence, we can exploit **related-key differential characteristics** to obtain the secret key **faster** than exhaustive search in the **single-key model**.

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Higher-order Differential Characteristic

For a function $f : S \to T$, the **derivative** at a **point** $a_1 \in S$ is defined as

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We can recursively define the i^{th} **derivate** of f as

$$\Delta_{a_1,\ldots,a_i} f(x) = \Delta_{a_i}(\Delta_{a_1,\ldots,a_{i-1}} f(x)).$$

Higher-order Differential Characteristic (Simplified)

Let us assume the values $a_1, ..., a_i$ be linearly independent.

$$\Delta_{a_1,...,a_i} f(x) = \sum_{c \in L(a_1,...,a_i)} f(x \oplus c),$$

where $L[a_1, \ldots, a_i]$ is the set of all 2^i possible linear combinations of $a_1, \ldots, a_i$.

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If the **algebraic degree** of a **function** f with respect to the input x is at most d, then

$$\Delta_{a_1,...,a_{d+1}} f(x) = \sum_{c \in L(a_1,...,a_{d+1})} f(x \oplus c) = 0$$

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- The **key space** can simply be **partitioned** into 2^{k-d-1} sets of the same size 2^{d+1}.
- We compute the corresponding ciphertexts for $2^{d+1} - 1$ keys in one set.

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- Obviously, this conventional technique **cannot** be exploited to **beat exhaustive search**.

Overview of our Technique

Target key x	Binary representation	Set of key generates the target key
0	0000	Guess
1	0001	Guess
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7	0111	4, 5, 6 free
8	1000	Guess

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3	0011	0, 1, 2 free
4	0100	Guess
5	0101	0, 1, 4 free
6	0110	0, 2, 4 free
7	0111	4, 5, 6 free
8	1000	Guess
9	1001	**0, 1, 8** free
10	1010	**0, 2, 8** free
11	1011	**1, 2, 8** free
12	1100	**0, 4, 8** free
13	1101	**1, 4, 8** free
14	1110	**2, 4, 8** free
15	1111	**12, 13, 14** free

Our Algorithm

Input: Known plaintext–ciphertext pair P, C
Output: Speeding up the exhaustive search.

1: **for** $x = 0$ to $2^k - 1$ **do**
2: **if** $wh(K_x) < d + 1$ **then**
3: Compute $C_x = E_x(P)$
4: **if** $C = C_x$ **then**
5: Return x
6: **end if**
7: **else**
8: **Consider $e_{i_1}, e_{i_2}, \ldots, e_{i_d}$ such that $c_{i_j} \neq 0$ in the binary representation of x**, for $0 \leq j \leq d$
9: Compute $C' = \bigoplus_{m \in L(e_{i_1}, e_{i_2}, \ldots, e_{i_d}, x) \setminus \{x\}} C_m$
10: **if** $C = C_x$ **then**
11: Return x
12: **end if**
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Complexity of the algorithm

- The **time complexity** of the mentioned algorithm is $\sum_{i=0}^{d} \binom{k}{i}$ full encryptions for the **guessed keys**.

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- The number of required **guessed keys** is surprising **small** but it requires **large memory** (order of 2^k).
- The cost of a memory access directly depends on the size of memory.

Modified Algorithm

Input: Known plaintext–ciphertext pair P, C

Output: Speeding up the exhaustive search.

1: **for** $x = 0$ to $2^k - 1$ **do**
2: **if** $wh(K_x) < d + 1$ OR $wh(K_{xt}) < d + 1$ **then**
3: Compute $C_x = E_{K_x}(P)$
4: **if** $C = C'$ **then**
5: Return x
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8: Consider $e_{i_1}, e_{i_2}, \ldots, e_{i_d}$ such that $c_{i_j} \neq 0$ in the binary representation of x, for $0 \leq j \leq d$
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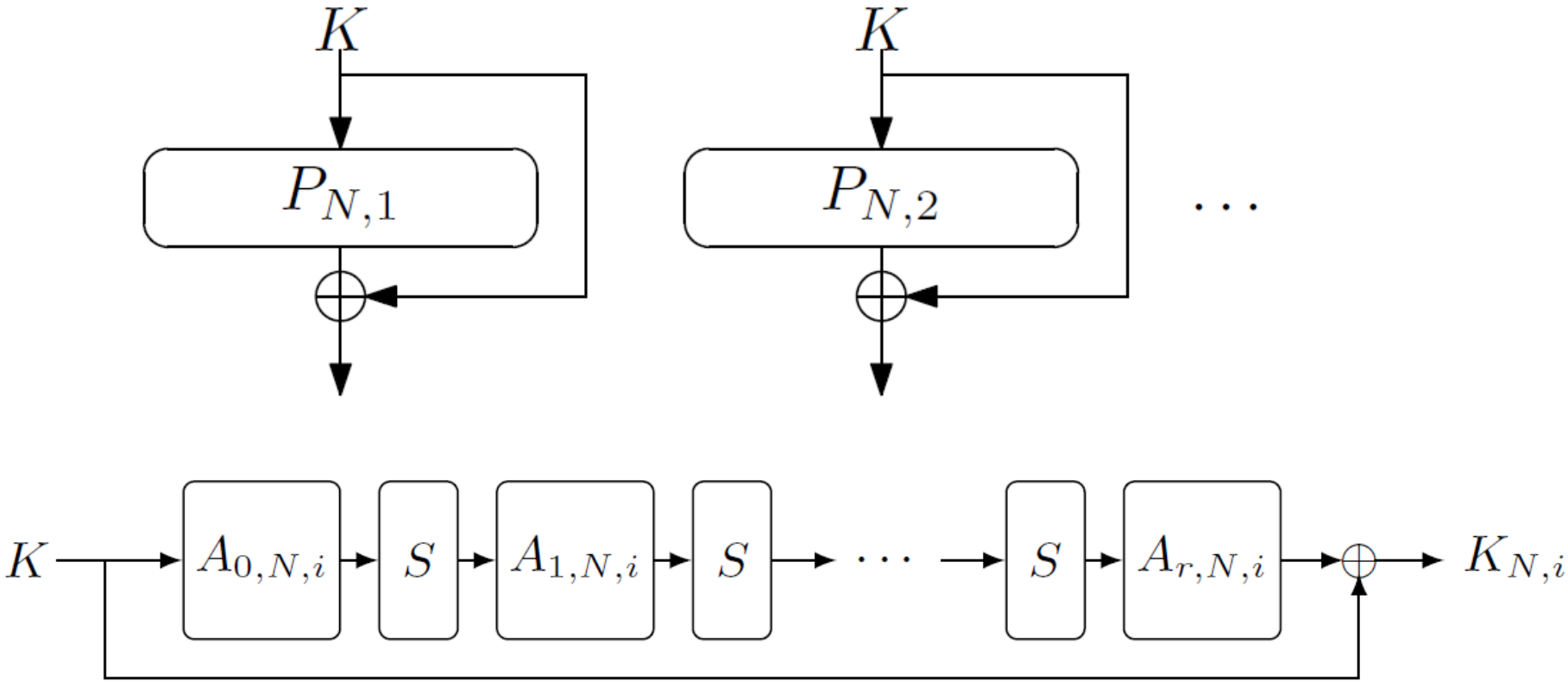
Rasta and Agrasta

Presented at CRYPTO 2018



Results on Agrasta

Rounds	Degree	Data	t	Time		Memory
				Full encryption	MA per each obtaining key	
r	2^r	1	t	$2^{k-t}\sum_{i=0}^{d}\binom{t}{i}$	2^d	$O(2^{t+1})$
1	2	1	8	$2^{k-2.8}$	2^2	$O(2^9)$
2	4	1	12	$2^{k-2.36}$	2^4	$O(2^{13})$
2	4	1	16	$2^{k-4.71}$	2^4	$O(2^{17})$
3	8	1	20	$2^{k-2.1}$	2^8	$O(2^{21})$
3	8	1	24	$2^{k-3.78}$	2^8	$O(2^{25})$
4	16	1	35	$2^{k-1.83}$	2^{16}	$O(2^{36})$
4	16	1	40	$2^{k-3.12}$	2^{16}	$O(2^{41})$

Conclusion

- The effect of related-key cryptanalysis in the single-key model has been a long-standing open question.
- In this work we present a general formalization for utilizing related-key higher-order differential characteristic to assess the precise security of cryptographic primitives in the single-key model.
- The effectiveness of our method directly depends on the degree of the function.